Modeling of Autonomous Hexa-Rotor Microcopter

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Abstract—Unmanned autonomous aerial vehicles have become a real center of interest. In the last few years, their utilization has significantly increased. During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multicopters. Today, they are used for multiple tasks such as navigation and transportation. This paper presents the development of a dynamic modeling and control algorithm—backstepping controller of an autonomous hexa-rotor microcopter. The autonomous hexa-rotor microcopter is an under-actuated and dynamically unstable nonlinear system. The model that represents the dynamic behavior of the hexa-rotor microcopter is complex. Unmanned autonomous aerial vehicles applications are commonly associated with exploration, inspection or surveillance tasks.

Keywords—dynamic model, dynamic behavior, unmanned autonomous aerial vehicles, autonomous hexa-rotor microcopter, under-actuated, dynamically unstable nonlinear system, control strategies, backstepping controller.

I. INTRODUCTION

Unmanned autonomous aerial vehicles have become a real center of interest [1-14]. In the last few years, their utilization has significantly increased. Today, they are used for civil and military applications, for multiple tasks such as navigation and transportation. Unmanned autonomous aerial vehicles applications are commonly associated with exploration, inspection or surveillance tasks.

During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multicopters [16, 18-19, 21-27]. One of the unmanned autonomous aerial vehicles with a strong potential is the hexa-rotor microcopter.

The autonomous hexa-rotor microcopter have numerous advantages over quadrotors, since they can offer more:
- more payload,
- the longest flight time and
- high maneuverability

The autonomous hexa-rotor microcopter, Figure 1, consists of six rotors attached to a rigid body frame and these additional two rotors make it able to carry:

- more payload,
- the longest flight time and
- high maneuverability

compared to quadrotor [24-29]. The autonomous hexa-rotor microcopters have additional redundancy over autonomous quad-rotor microcopters.

The control design carried out for an autonomous quad-rotor microcopter can be applied to the autonomous hexa-rotor microcopter since they are modeled as a rotating rigid body dynamic system with six degrees of freedom (6 DOF), Figure 2.

Figure 1. Hexa-rotor microcopter used for the experiments

Figure 2. The Hexa-rotor microcopter in hovering conditions
The autonomous hexa-rotor microcopter is an:
- under-actuated and
- dynamically unstable nonlinear system.

The model that represents the dynamic behavior of the hexa-rotor microcopter is nonlinear and complex. This paper presents the development of a dynamic modeling and control algorithm of an autonomous hexa-rotor microcopter.

The paper is organized as follows:

Section 1: Introduction.
In Section 2, the dynamic modeling of a hexa-rotor microcopter is presented.
In Section 3, backstepping controller for hexa-rotor microcopter is presented.
Conclusions are given in Section 4.

II. DYNAMIC MODELING OF HEXA-ROTOR MICROCOPTER

The model of the hexa-rotor helicopter and the rotational directions of the propellers are presented in Figure 3. This cross structure is quite thin and light, however it shows robustness by linking mechanically the motors [9], [20], [30]. Hexa-rotor microcopter body is rigid. The six rotors are symmetrically distributed around the center. All the propeller axes of rotation are fixed and parallel. Propellers are rigid. These considerations point out that the structure is quite rigid and the only things that can vary are the propeller speeds.

The hexa-rotor microcopter configuration has six rotors which generate the propeller forces $F_i$ ($i = 1, 2, 3, 4, 5, 6$) as it is shown in Figure 3. Control of quadrotor is achieved by commanding different speeds to different propellers, which in turn produces differential aerodynamic forces and moments. In order to increase the altitude of the aircraft it is necessary to increase the rotor speeds altogether with the same quantity [9].

Forward motion is accomplished by increasing the speed of the rotors (3, 4, 5) while simultaneously reducing the same value for forward rotors (1, 2, 6).

For leftward motion the speed of rotors (5 and 6) is increased while the speed of rotors (2 and 3) is reduced.

Backward and rightward motion can be accomplished similarly. Finally, yaw motion can be performed by speeding up or slowing down the clockwise rotors depending on the desired angle direction.

To describe the motion of a 6 DOF rigid body it is usual to define two reference frames (Fig. 2):
- the earth inertial frame (E-frame), and
- the body-fixed frame (B-frame).

The equations of motion are formulated using the Newton-Euler laws with the following reasons:
- the inertia matrix is time-invariant;
- advantage of body symmetry can be taken to simplify the equations;
- measurements taken on-board are easily converted to body-fixed frame;
- control forces are almost always given in body-fixed frame.

The E-frame ($O^E_{xyz}$) is chosen as the inertial right hand reference. This frame is used to define the linear position (in meters) and the angular position (in radians) of the quadrotor.

The B-frame is attached to the body. The origin of the B-frames is chosen to coincide with the center of the hexa-rotor microcopter cross structure. This reference is right-hand, too.

The linear position of the helicopter (X, Y, Z) is determined by the coordinates of the vector between the origin of the B-frame and the origin of the E-frame according to equation.

The angular position of the hexa-rotor microcopter ($\Phi$, $\theta$, $\psi$) is defined by the orientation of the E-frame with respect to the E-frame. This is given by three consecutive rotations about the main axes which take the E-frame into the B-frame. In this paper, the "roll-pitch-yaw" set of Euler angles ($\Phi$, $\theta$, $\psi$) were used.

The vector that describes quad-rotor position and orientation with respect to the E-frame can be written in the form:

$$s = [x, y, z, \Phi, \theta, \psi]^T$$

(1)

The rotation matrix between the E-frame and B-frames has the following form:

$$R = \begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & \sin \theta \\
\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \cos \theta \\
-\cos \phi \sin \theta \cos \psi - \sin \phi \sin \psi & -\cos \phi \sin \theta \sin \psi + \sin \phi \cos \psi & \cos \phi \cos \theta
\end{bmatrix}$$

(2)

Now, the model of hexa-rotor dynamics can be described by a system of equations:
\[
\dot{x} = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} + A_{10}
\]
\[
\dot{y} = (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{U_1}{m} + A_{12}
\]
\[
\dot{z} = -g + \cos \theta \cos \phi \frac{U_1}{m} + A_8
\]
\[
\dot{\phi} = \frac{I_y - I_z}{I_x} \dot{\psi} - \frac{J_R}{I_x} \dot{\theta} \omega + \frac{U_2}{I_x} d + A_2
\]
\[
\dot{\theta} = \frac{I_z - I_y}{I_x} \dot{\phi} \psi - \frac{J_R}{I_y} \dot{\theta} \phi + \frac{U_3}{I_y} d + A_4
\]
\[
\dot{\psi} = \frac{I_x - I_z}{I_z} \dot{\phi} \hat{\phi} + \frac{U_4}{I_z} + A_6
\]

### III. BACKSTEPPING CONTROLLER FOR HEXA-ROTOR MICROCOPTER

In this paper, controller design for the hexa-rotor microcopter is proposed by using backstepping technique. Backstepping is a recursive design methodology that makes use of Lyapunov stability theory to force the system to follow a desired trajectory. The hexa-rotor microcopter is controlled by angular speeds of six motors. Each motor produces a thrust and a torque, whose combination generates the main thrust, the yaw torque, the pitch torque, and he roll torque acting on the hexa-rotor microcopter. First, the dynamical model is rewritten in state-space form:

\[
\dot{X} = f(X, U)
\]

by introducing:

\[X = [x_1, x_2, y]^T \in \mathbb{R}^{12}\]

as space vector of the system:

\[x_1 = \phi \quad x_3 = \psi \quad x_5 = Y\]
\[x_2 = \dot{x}_1 \quad x_6 = \dot{x}_3 \quad x_{10} = \dot{x}_9 = \dot{Y}\]
\[x_3 = \theta \quad x_7 = X \quad x_{11} = Z\]
\[x_4 = \dot{x}_3 \quad x_8 = \dot{x}_7 \quad x_{12} = \dot{x}_{11} = \dot{Z}\]

Next, the x-coordinates are transformed into the new z-coordinates:

\[z_1 = x_{1_{\text{ref}}} - x_1 \quad z_7 = x_{7_{\text{ref}}} - x_7\]
\[z_2 = x_2 - x_{1_{\text{ref}}} - \alpha_1 z_1 \quad z_8 = x_8 - x_{7_{\text{ref}}} - \alpha_7 z_7\]
\[z_3 = x_{3_{\text{ref}}} - x_3 \quad z_9 = x_{9_{\text{ref}}} - x_9\]
\[z_4 = x_4 - x_{3_{\text{ref}}} - \alpha_3 z_3 \quad z_{10} = x_{10} - x_{9_{\text{ref}}} - \alpha_9 z_9\]
\[z_5 = x_{5_{\text{ref}}} - x_5 \quad z_{11} = x_{11_{\text{ref}}} - x_{11}\]
\[z_6 = x_6 - x_{5_{\text{ref}}} - \alpha_5 z_5 \quad z_{12} = x_{12} - x_{11_{\text{ref}}} - \alpha_{11} z_{11}\]

By introducing the partial Lyapunov functions [15], [17], [31] to all x-coordinates results in the following backstepping controller:

\[U_x = \frac{m}{U_1} (z_3 - \alpha_1 (z_{10} + \alpha_7 z_7) - \alpha_3 z_3)\]
\[U_y = \frac{m}{U_1} (z_6 - \alpha_5 (z_{10} + \alpha_9 z_9) - \alpha_3 z_3)\]
\[U_z = \frac{m}{\cos x_1 \cos x_3} (z_{12} + g - \alpha_1 (z_{10} + \alpha_7 z_7) - \alpha_3 z_3)\]

The position of the hexa-rotor microcopter in the earth reference frame is illustrated in Figure 4.

![Figure 4. Position of the hexa-rotor microcopter in the earth reference frame.](image)

### IV. CONCLUSIONS

This paper presents the development of a dynamic modeling and control algorithm of an autonomous hexa-rotor microcopter. During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multicopters.

The autonomous hexa-rotor microcopter is an underactuated and dynamically unstable nonlinear system. The model that represents the dynamic behavior of the hexa-rotor microcopter is complex. Unmanned autonomous aerial vehicles have become a real center of interest. In the last few years, their utilization has significantly increased.

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